# DSP 연습문제 풀이

고려대학교 컴퓨터정보통신대학원 전희원

2.5

(a) $\cos(\theta_1+\theta_2) = \cos(\theta_1)\cdot\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

sol) Use Inverse Euler Formulas

$$= \frac{(e^{j\theta_1}+e^{-j\theta})}{2}\cdot\frac{(e^{j\theta_2}+e^{-j\theta_2})}{2} + \frac{(e^{j\theta_1}-e^{-j\theta_1})(e^{j\theta_2}-e^{-j\theta_2})}{4}$$

$$= \frac{2e^{j(\theta_1+\theta_2)} + 2e^{-j(\theta_1+\theta_2)}}{4} = \frac{e^{j(\theta_1+\theta_2)}+e^{-j(\theta_1+\theta_2)}}{2}$$

$$\therefore \cos(\theta_1+\theta_2) = \frac{e^{j(\theta_1+\theta_2)}+e^{-j(\theta_1+\theta_2)}}{2}$$

$$\therefore \cos(\theta_1+\theta_2) = \cos(\theta_1)\cdot\cos(\theta_2) - \sin(\theta_1)\cdot\sin(\theta_2)$$

(b) $\cos(\theta_1-\theta_2) = \cos\theta_1\cdot\cos\theta_2 + \sin(\theta_1)\cdot\sin(\theta_2)$

sol) Use Inverse Euler Formulas

$$= \frac{(e^{j\theta_1}+e^{-j\theta_1})\cdot(e^{j\theta_2}+e^{-j\theta_2})}{4} - \frac{(e^{j\theta_1}-e^{-j\theta_1})(e^{j\theta_2}-e^{-j\theta_2})}{4}$$

$$= \frac{\cancel{e^{j(\theta_1+\theta_2)}} + e^{j(\theta_1-\theta_2)} + e^{-j(\theta_1-\theta_2)} + \cancel{e^{-j(\theta_1+\theta_2)}}}{4} - \frac{\cancel{e^{j(\theta_1+\theta_2)}} - e^{j(\theta_1-\theta_2)} - e^{-j(\theta_1-\theta_2)} + \cancel{e^{-j(\theta_1+\theta_2)}}}{4}$$

$$= \frac{\cancel{2}(e^{j(\theta_1-\theta_2)}+e^{-j(\theta_1-\theta_2)})}{\cancel{4}\,2} = \frac{e^{j(\theta_1-\theta_2)}+e^{-j(\theta_1-\theta_2)}}{2}$$

$$= \cos(\theta_1-\theta_2)$$

2.9 (a) to : $x(t) = A\cos(\omega_0 t + \phi)$

$x(t) = 2\cos(\omega_0 t - 45^\circ) + \cos(\omega_0 t)$

$= 2(\cos(\omega_0 t)\cos(45^\circ) + \sin(\omega_0 t)\sin(-45^\circ)) + \cos(\omega_0 t)$

$= 2(\cos(\omega_0 t)\underbrace{\cos(45^\circ)}_{\frac{\sqrt{2}}{2}} - \sin(\omega_0 t)\underbrace{\sin(45^\circ)}_{\frac{\sqrt{2}}{2}}) + \cos(\omega_0 t)$

$= (\sqrt{2}\cos(\omega_0 t) - \sqrt{2}\sin(\omega_0 t)) + \cos(\omega_0 t)$

$= (1+\sqrt{2})\cos(\omega_0 t) - \sqrt{2}\sin(\omega_0 t)$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\alpha = \omega_0 t$ 로 하면

$\cos\beta = 1+\sqrt{2}$ , $\sin\beta = \sqrt{2}$

$\tan^{-1}\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right) = 30.36^\circ$

$\therefore x(t) = \cos(\omega_0 t + 30.36^\circ)$

(b) period $= \frac{2\pi}{5\pi} = \frac{2}{5}$ (sec) $= 0.4$ (sec)

$0.4 \times 5 = 2$sec , 5번

$0.4 \times 2 = 0.8$ sec  0.8 sec ~ 2sec 사이에

1번의 period가 존재

(c) $\cos(\omega_0 t + 30.36^\circ) = \frac{e^{j(\omega_0 t + 30.36)} + e^{-j(\omega_0 t + 30.36)}}{2}$

$= \frac{e^{j(\omega_0 t)}e^{j(30.36)} + e^{-j(\omega_0 t)} \cdot e^{-j(30.36)}}{2}$  $X = \text{phase} = e^{j(30.36)}$

$= \frac{1}{2}X e^{j\omega_0 t} + \frac{1}{2}X^* e^{-j(\omega_0 t)}$ , $\bar{x}(t) = \frac{1}{2}X e^{j(\omega_0 t)}$

2.14 $A\cos(\omega_0 t + \phi) = \mathrm{Re}\{Ae^{j\phi}e^{j\omega_0 t}\}$

( Amplitude : 5
$\phi$ : $-0.8\pi$
$\omega_0$ : $2\pi(158)$ )

---

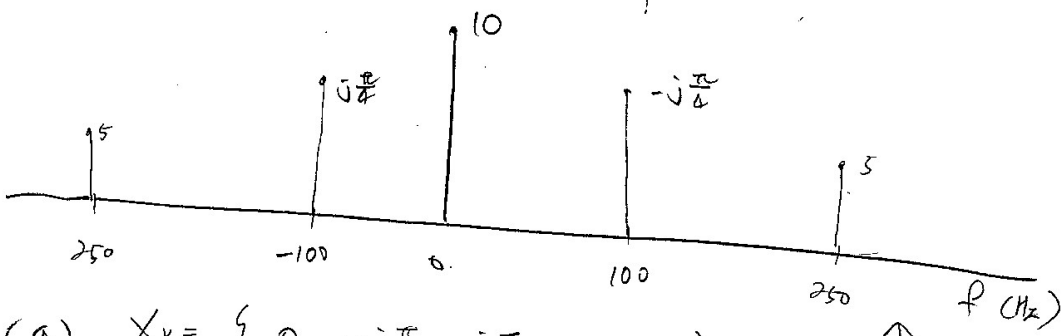
3.5 $x(t) = 10 + 20\cos(2\pi(100)t + \pi/4) + 10\sin(2\pi(250)t)$

$= 10 + 10\left(e^{j\frac{\pi}{4}} \cdot e^{j2\pi(100)t} + e^{-j\frac{\pi}{4}} \cdot e^{-j(2\pi(100)t)}\right)$

$+ 5\left(e^{j(2\pi(250)t)} + e^{-j(2\pi(250)t)}\right)$

$\{(10, 0), (10e^{j\frac{\pi}{4}}, 100), (10^{-j\frac{\pi}{4}}, -100)$

$, (5, 250), (5, -250)\}$




(a) $X_k = \{0, -j\frac{\pi}{4}, j\frac{\pi}{4}, 5, 5\}$
phase

(b) $T_1 = \frac{1}{100}$, $T_2 = \frac{1}{250}$ → 정현 periodic

(c)

$l_1 T_1 = l_2 T_2$

$l_1 T_1 = 2 \cdot \frac{1}{100} = \frac{1}{50}$ (sec) $\quad \frac{T_1}{T_2} = \frac{l_2}{l_1} = \frac{\frac{1}{100}}{\frac{1}{250}} = \frac{5}{2}$

common period $\quad l_2 = 5, \; l_1 = 2$

3.9. $x(t) = 2 + 4\cos(40\pi t - \pi/5) + 3\sin(60\pi t) + 4\cos(120\pi t - \pi/3)$

(a) $f_1 = 20, \quad f_2 = 30, \quad f_3 = 60$

$T_1 = \frac{1}{20}, \quad T_2 = \frac{1}{30}, \quad T_3 = \frac{1}{60}$

$\underline{\underline{l_1 T_1 = l_2 T_2}} = l_3 T_3$

$\frac{l_2}{l_1} = \frac{T_1}{T_2} = \frac{30}{20} = \frac{3}{2} \quad \therefore \underline{\underline{l_1 = 2}}$

$l_1 T_1 = T_0, \quad 2 \cdot \frac{1}{20} = \frac{1}{10}$

$T_0 = \frac{1}{10}$ (sec). $\quad \omega_0 = 10$ (Hz)

$2 + 2(e^{-j\pi/5} \cdot e^{j40\pi t} + e^{-j40\pi t} e^{j\pi/5})$

$+ \frac{3}{2}(e^{-j\pi/2} \cdot e^{j60\pi t} + e^{-j60\pi t} e^{j\pi/2})$

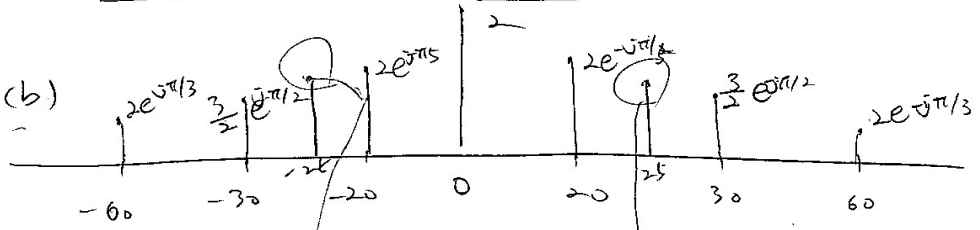
$+ 2(e^{-j\pi/3} \cdot e^{j120\pi t} + e^{-j120\pi t} \cdot e^{j\pi/3})$

$(2, 0), \quad (\underline{2e^{-j\pi/5}}, 20), \quad (\underline{2e^{j\pi/5}}, -20)$

$(\underline{\frac{3}{2}e^{-j\pi/2}}, 30), \quad (\underline{\frac{3}{2}e^{j\pi/2}}, -30)$

$(\underline{2e^{-j\pi/3}}, 60), \quad (\underline{2e^{j\pi/3}}, -60)$

(b)




(c) $10\cos(50\pi t - \pi/6)$ $\quad f_4 = 25, \quad T_4 = \frac{1}{25}$

$l_4 T_4 = \frac{1}{10}, \quad l_4 = \frac{1}{10} \cdot \frac{1}{25} = \frac{1}{250}$ $\quad$ non periodic

$5(e^{-j\pi/6} \cdot e^{j50\pi t} + e^{-j50\pi t} e^{j\pi/6})$

$(5e^{-j\pi/6}, -25), \quad (5e^{j\pi/6}, 25)$